Backpropagation for Parametric Signal Temporal Logic

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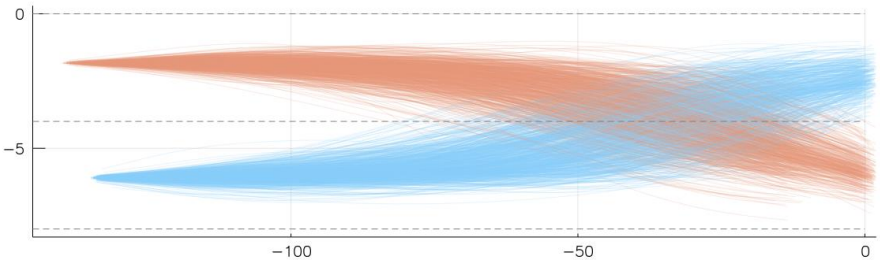
How can we characterize and quantify a driving scenario?

Use Parametric Signal Temporal Logic

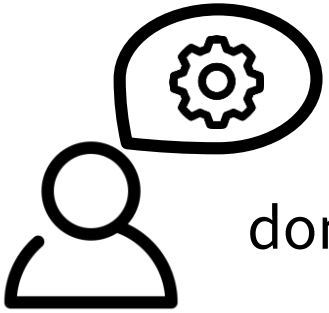
How do we make this computationally tractable?

Use computation graphs and off-the-shelf machine learning software

Driving scenario



Time-series data



Human
domain-knowledge

Signal Temporal Logic

- Specification language
- Measure of satisfaction

STL grammar

$$I := (a,b) \mid (a,b] \mid [a,b) \mid [a,b]$$

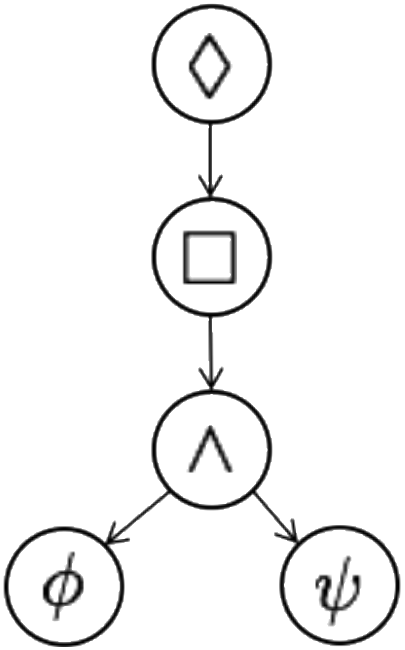
$$\phi := true \mid f(s) < c \mid \text{Not} \mid \text{And} \mid \text{Or}$$

Eventually | Always | Until | Then

$$\Diamond_I \Box (\psi \wedge \phi)$$

$$\psi = s > 2$$

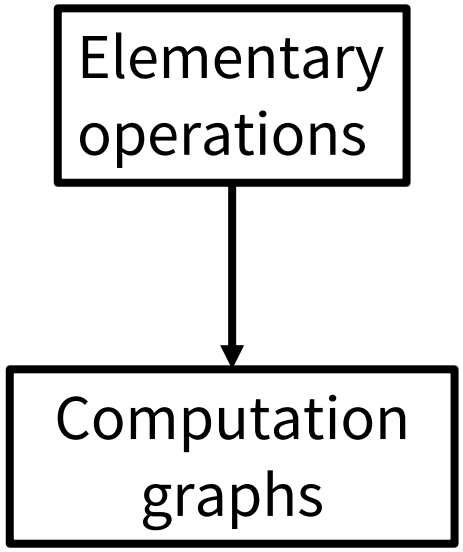
$$\phi = s < 5$$

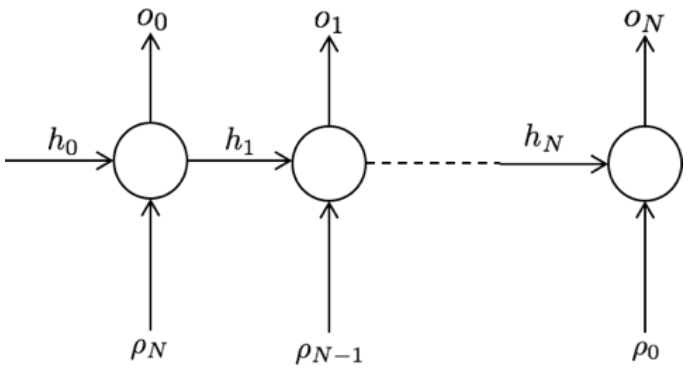


Evaluating STL formulas

Robustness formulas

$$
\begin{aligned}
\rho(s,t,true) &= \rho_{\max} \\
\rho(s,t,f(s)<c) &= c - f(s(t)) \\
\rho(s,t,\neg\phi) &= -\rho(s,t,\phi) \\
\rho(s,t,\phi\wedge\psi) &= \min(\rho(s,t,\phi),\rho(s,t,\psi) \\
\rho(s,t,\phi\vee\psi) &= \max(\rho(s,t,\phi),\rho(s,t,\psi) \\
\rho(s,t,\Diamond_I\phi) &= \max_{t'\in I\oplus t}\rho(s,t',\phi) \\
\rho(s,t,\Box_I\phi) &= \min_{t'\in I\oplus t}\rho(s,t',\phi) \\
\rho(s,t,\phi\mathcal{U}_I\psi) &= \max_{t'\in I\oplus t}(\min(\rho(s,t',\psi), \\
&\qquad \min_{t''\in[t,t']}\rho(s,t'',\phi))) \\
\rho(s,t,\phi\mathcal{T}_I\psi) &= \max_{t'\in I\oplus t}(\min(\rho(s,t',\psi), \\
&\qquad \max_{t''\in[t,t']}\rho(s,t'',\phi)))
\end{aligned}
$$





What if you do not know the formula precisely?

Use a template formula and learn the parameters

$$\square\ f(s) < \alpha$$

Parametric STL

$\square f(s) < \alpha$

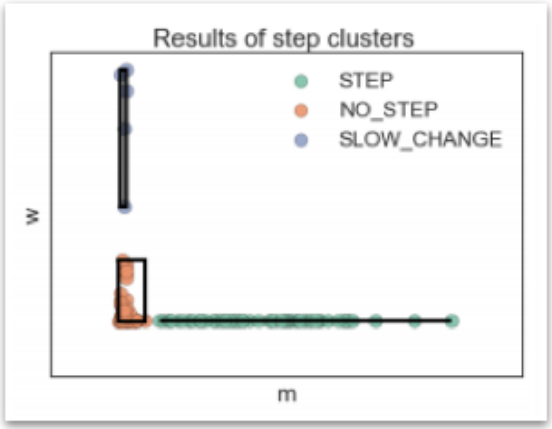
pSTL

Optimizer

$\alpha_1, \alpha_2, \ldots$

Features

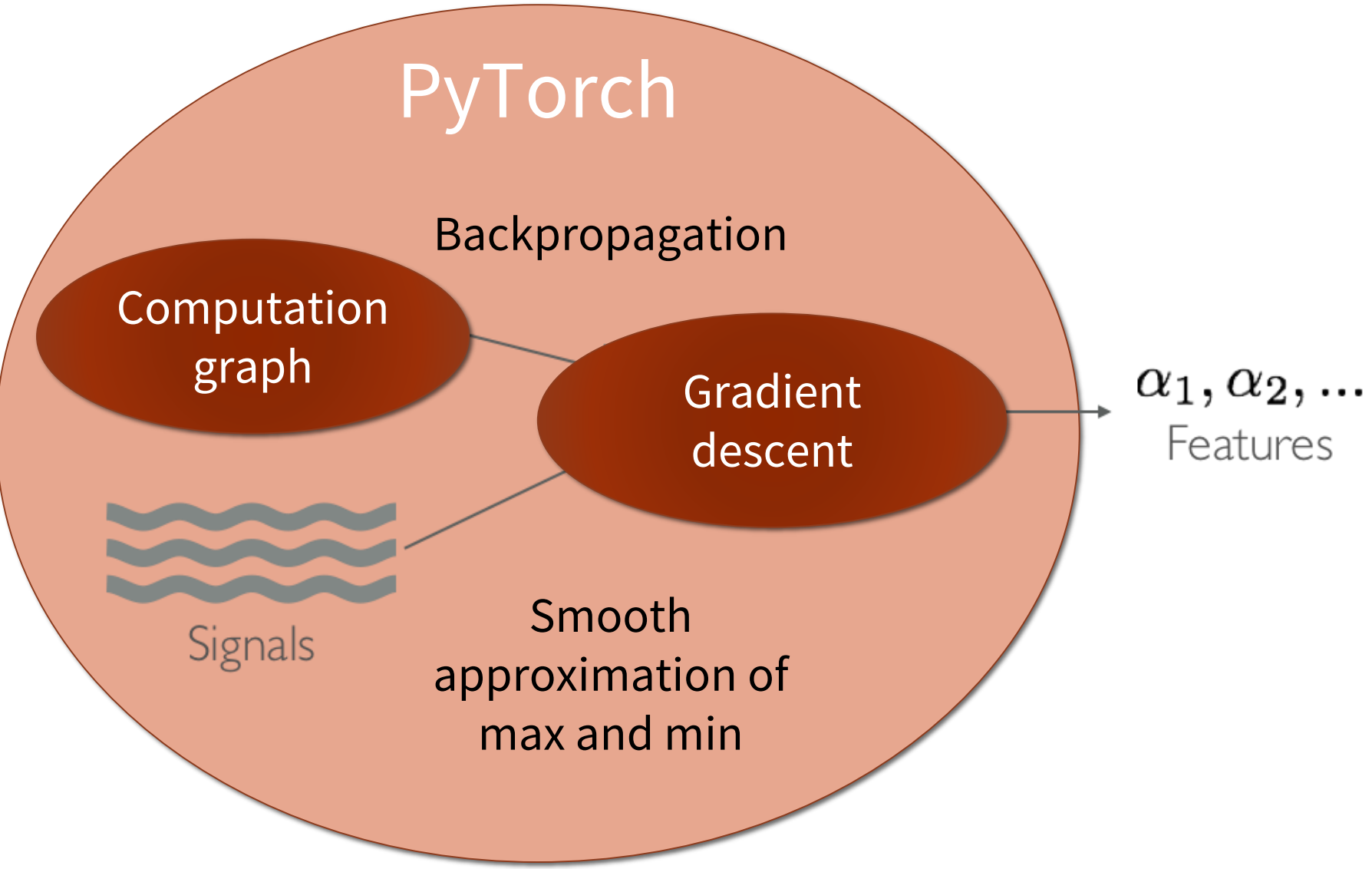
Signals



Machine Learning (e.g., clustering)

M. Vazquez-Chanlatte, J. V. Deshmukh, X. Jin, and S. A. Seshia, "Logical clustering and learning for time-series data," Computer Aided Verification , vol. 10426, pp. 305–325, 2017.

A. Bakhirkin, T. Ferre`re, and O. Maler, "Efficient parametric identifi- cation for STL," in Hybrid Systems: Computation and Control, 2018.

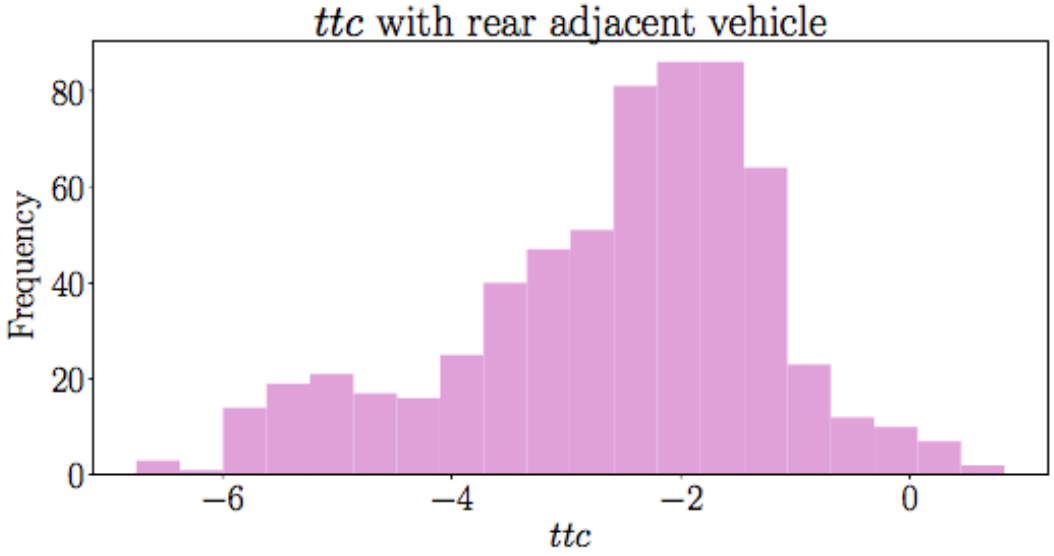


https://github.com/StanfordASL/stlcg

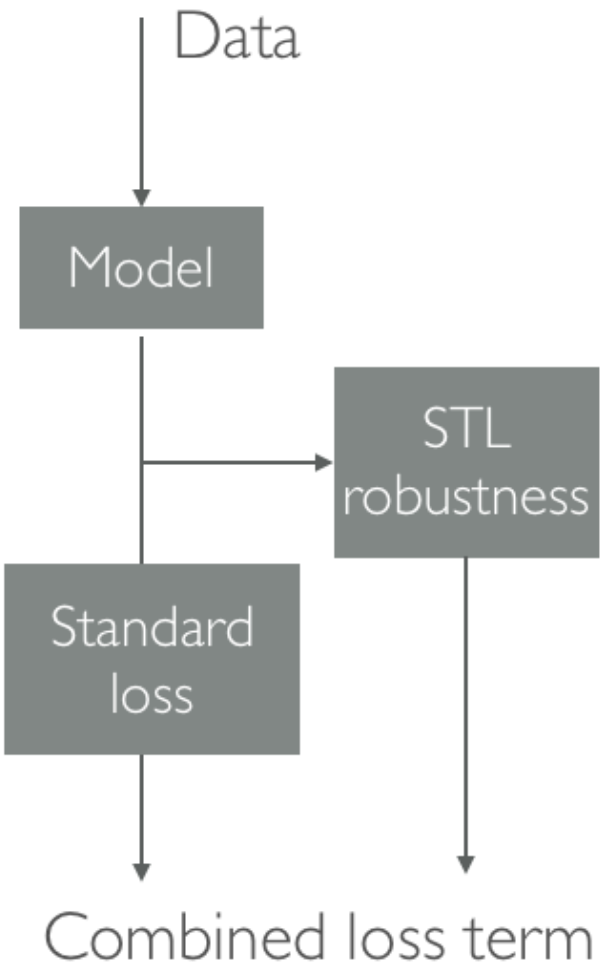
Case Study

$$ttc(x_1, x_2, v_1, v_2) = -\frac{x_1 - x_2}{v_1 - v_2}$$

$$\phi = in\_start\_lane\ \mathcal{U}\ ttc < \alpha$$



Conclusions



- STL can describe properties of time-series data
- Efficient way to evaluate the robustness of formulas using computation graphs
- Combine STL with existing frameworks
  - Improve performance
  - Enforce structure
  - Ensure feasibility