Offline and Online Tyre Model Reconstruction by Locally Weighted Projection Regression

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***Abstract*—This paper provides an analysis of methods used in automotive control applications for finding the tyre forces. An attention is given to three main classes of relevant methods: tyre-model-based, tyre-model-free, and sensor-based. After analysis of advantages and disadvantages of each class, an original application of the approach based on locally weighted projection regression (LWPR) is discussed. This approach can find combined use for both model-free and sensor-based tyre force reconstruction.**

***Index Terms*—tyre, vehicle dynamics, vehicle control, tyre model, locally weighted projection regression**

I. Overview of Methods for Tyre Force Reconstruction

A. Introduction

The problem of tyre force reconstruction belongs to one of the most important tasks by designing the vehicle motion control systems. To ensure proper control on the vehicle safety, comfort, driving efficiency and other functions, corresponding on-board systems should handle in real-time the information about longitudinal, lateral and vertical tyre forces. This can be done either by use of corresponding state observers or by direct measurement of tyre forces. The latest option is definitely more advantageous for the system design from practical viewpoint. However, sensor technologies for tyre forces and torques have still various technological obstacles for the use on mass-production vehicles. Available solutions in this area are therefore mainly implemented on experimental and test vehicles. As a result, the observation remains as the main tool for tyre force reconstruction in the automotive controllers. Here two main techniques can be identified: tyre-model-based and tyre-model-free. Next sections provide an overview for the most typical solutions for each class of the methods.

B. Tyre-model-based Force Reconstruction

The tyre-model-based force reconstruction represents a virtual tyre sensor, which observer is using one or another tyre models. Considering requirements to real-time operation of virtual tyre sensors, highly precise and complex tyre models are finding less application here, and the priority is given to semi-empirical and sufficiently simplified physical models of tyre-surface interaction. As applied to longitudinal and lateral tyre forces, widely-accepted techniques cover predominantly the Dugoff tyre model (linear and non-linear variants) [1], [2], [3] and the Magic Formula tyre model [4]. Few studies are also proposed the solutions with other models of different complexity as the Burckhardt model [5], the *arctan*-function-based model [6], and TMeasy [7].

In most cases the variations of the Kalman filter (extended, unscented and dual) are applied as the corresponding estimation tools. They require a priori knowledge about relevant tyre states as tyre longitudinal slip, tyre lateral slip, vertical load etc. This causes certain limitations of this method. Firstly, not all the states can be directly measured with the conventional automotive on-board systems and, as a result, extra estimators can be required, e.g. for the slip-related parameters. Secondly, tyre forces are influenced by many operational factors, for instance, by tyre inflation pressure, contact temperature, and road surface roughness. Despite the consideration of these and another factors reduces the tyre model uncertainty, this increases the model complexity.

C. Tyre-model-free Force Reconstruction

The limitations of the previous class have motivated many studies, where a tyre model is not required for the force reconstruction. One of the main ideas behind such tyre-model-free approach is to emulate the tyre forces as random variables. These variables can then be embedded into a model-based state estimator of the vehicle planar dynamics, which is used by the vehicle motion controller.

The work [8] proposes to identify three methods for building the tyre-model-free reconstruction of the longitudinal force: based on vehicle dynamics, based on wheel rotation dynamics, and stochastic. The methods based on vehicle dynamics use predominantly the vehicle acceleration sensor information and GPS signals to derive the forces from the longitudinal force balance equations with consideration of driving resistance parameters [9], [10]. The methods based on the wheel rotation dynamics require the information from the wheel speed sensors to reconstruct the longitudinal tyre force, for example, from wheel torque balance equations. The corresponding examples can be found in [11], [12] and

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[13]. Finally, by the stochastic methods, the forces are mainly interpreted as random-walk or "black-box" variables and can be derived using different vehicle planar models [14], [15]. As for the lateral force reconstruction, both stochastic and vehicle-dynamics-based methods can be also applied.

The tyre-model-free force reconstruction is especially beneficial in the case of uncertain road friction parameters because an estimation of a corresponding friction scaling factor is not required to correct the reconstructed tyre forces. Nevertheless, this approach can have limitations in terms of complex tuning and computations costs.

D. Sensor-based Force Reconstruction

The force sensing technology has been intensively studied for the last decades to develop accurate, robust and inexpensive solution. Due to the fact that transmission of forces and moments affects all components between tire road contact and vehicle body, all components carrying the load can be used to force/torque measurement and reconstruction, e.g. tyre, rim, bearing and suspension. Several approaches and their limitations are discussed below:

a) Suspension bushing deformation: the forces transmitted through the suspension bushing can be reconstructed via direct deformation measurement using eddy-current displacement sensors [16] or the estimation of the relative bushing deformation based on acceleration measurement [17]. The main drawbacks are complexity of the approach and the durability of the bushings.

b) Deformation between knuckle and brake calipers: this approach is based on the installation of a sensor bracket with strain resistance elements between knuckle and brake calipers to measure brake torque [18]. Only brake torque can be reconstructed in such approach and the method performance is temperature dependent.

c) Wheel force transducer: force/torque measurement is performed by strain measurements in the wheel rim [19]. Only this method is currently applied for commercial products (Kistler, MTS, Michigan Scientific Corporation, etc.) Although it provides high accuracy and bandwidth, the application of wheel force transducer as a standard vehicle sensor is too expensive even for premium class vehicles.

d) Tyre sidewall deformation: tyre deformation measurement can be used to reconstruct force using an optical position detection sensor [20], laser-based sensor system [21], a passive surface acoustic wave sensor [22] or by combination of a Hall sensor and magnet [16]. The common drawback of these approaches is the necessary adjustment and calibration after tyre replacement.

e) Tyre inner liner accelerometer: a MEMS accelerometer is located and fixed to the inner liner of the tyre [23]. The reconstruction of longitudinal and normal forces is demonstrated both in laboratory and road test conditions [24]. Furthermore also the measurement of lateral contact forces was recently demonstrated [25]. The common limitations are discontinuous measurement signal and durability of the approach due to the relatively short lifetime of tyres.

f) Bearing displacement or deformation based: Using the wheel-end bearing two principally different approaches can be applied. The first approach is displacement based (relative inner- to outer-ring displacement) using Hall effect [26], eddy-current sensors [27] or capacitive [28] sensors. Its limitation is that a limited number of loads can be reconstructed. The second approach is to measure outer-ring deformation using strain gauges [29], [30]. The measured strain should be translated to the bearing loading using empirical methods, e.g. least squares fitting or artificial neural networks. However, due to nonlinear behaviour, such translation is nontrivial and significantly affects accuracy of force reconstruction. Instead of empirical methods, the model-based approach for the estimation of bearing forces is proposed [31] using a cascaded extended and unscented Kalman filtering. An experimental study covering both laboratory and field tests showed that the model-based approach led to accurate load estimates in various conditions and outperforms the data-driven methods.

E. Summarising Remarks

The introduced short overview of basic approaches for the tyre force reconstruction demonstrates a variety of tools, which can be used in vehicle motion control systems. Taking into account such factors as uncertainties of tyre models as well as for demand on extensive test procedures for proper parameterisation of tyre models, the model-free reconstruction can be considered as a more advantageous candidate. An interesting advancement can be proposed in the case of development of hybrid approaches, where the same analytical base can be used both for the model-free and sensor-based reconstruction. Here it makes sense to apply not only conventional stochastic methods but also another variants of computational intelligence tools. One example of such an approach is discussed in next section.

II. Case study : Locally weighted projection regression

Locally weighted projection regression (LWPR) is an algorithm that supports non-linear function approximation in high dimensional spaces [32]. The nonlinear system behaviour, especially steady state, can be accurately captured by using this technique.

The key idea of this method is to approximate non-linear functions by using piece-wise linear models. The features of LWPR are numerical robustness in high dimensional spaces and the capability to perform incremental online learning with the predefined learning rate.

A. The LWPR algorithm

A weighting kernel used to determine the locality is defined in the way that computes a weight $w_{k,i}$ for each data point (x_i, y_i) corresponding to the distance from the centre c_k of the kernel within each local unit. Usually, a gaussian kernel is chosen,

$$w_{k,i} = exp(-\frac{1}{2}(x_i - c_k)^T D_k(x_i - c_k)), \tag{1}$$

where D_k is the distance metric that influences the size and shape of the region of validity or receptive field (RF). It is assumed that there are K locally linear models that are combined to form the prediction. Each linear model calculates a prediction y_k given an input vector x. The net output is the weighted mean of all the linear models.

Algorithm 1 shows how an incrementally locally weighted variant of partial least squares (PLS) is used to generate linear model parameters within the LWPR scheme. In the algorithm 1, $\lambda \in [0, 1]$ is the forgetting factor that decides amount of the old data of the parameters used in the regression will be forgotten. The PLS predictor adds linear projections in an incremental fashion until the point where adding further projections does not improve the accuracy.

The distance metric D influences the shape and size of each RF and thus also influences the effectiveness of each local model. This distance metric is optimized separately for each RF using an incremental gradient descent based on stochastic leave-one-out cross validation criterion. This is shown in the algorithm 2.

An incremental learning system which embeds the above update laws, and generates additional locally linear models as and when needed is shown in algorithm 3.

B. Current state of the art

The LWPR algorithm has been used to learn the dynamic model of a robot manipulator [33]. In comparison to other classical learning controllers, it was reported that the LWPR provides best performance when there is no a-priori knowledge of the system dynamics. The application of this algorithm for real-time robot learning has been presented in [34]. The results demonstrates the successful application of autonomous learning to complex robotic system. It was also concluded that this technique, using its learning abilities outperforms traditional control techniques. Comparison of the LWPR with a few other regression techniques to estimate the inverse dynamic model of a robot from measured data is demonstrated in [35]. This is mainly done in order to capture non-linearities arising from dynamics of hydraulic cables, actuator dynamics or complex friction dynamics.

Regarding automotive domain, this method has been only applied for the scaled off-road vehicle [36]. Based on the authors' knowledge, this paper is the first application of LWPR for tyre force reconstruction. Taking into account the above-mentioned advantages, it can be an interesting candidate for tyre force reconstruction, especially, regarding steady-state tyre properties. To evaluate the LWPR capability for tyre force reconstruction, two main features should be discussed: (i) capability to reconstruct tyre force based on offline training similar to data-driven learning techniques required a large data set; (ii) capability to learn tyre characteristic online assuming a repeatable track, e.g. racing laps. The following discussion is organized according to these features.

C. Offline learning

1) Pure longitudinal force reconstruction: The algorithm is presented with training data comprising of the longitudinal force F_x behaviour with longitudinal slip κ. See Table I for the parameters set during training. The longitudinal slip is set to $\kappa \in \begin{bmatrix}-0.5 & 0.5\end{bmatrix}$ and the algorithm is trained with the corresponding longitudinal force behaviour obtained from the baseline Delft-tyre model. The LWPR model is presented with test data and the result of reconstruction is shown in Fig. 1.

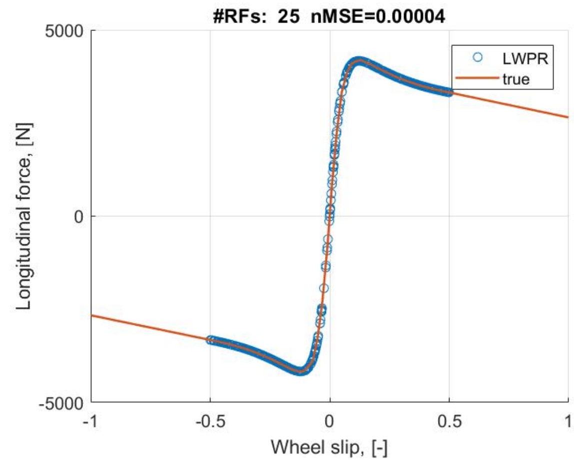

Fig. 1. Longitudinal force reconstruction $F_x(\kappa)$)

TABLE I
PARAMETERS FOR LONGITUDINAL FORCE RECONSTRUCTION

Parameter	Value	Unit
Normal load	4000	N
Initial distance metric (LWPR)	5000	[-]

2) Combined force reconstruction: The combined force behaviour with longitudinal slip κ, slip angle α and normal load F_z is presented as the training data set to the algorithm. The corresponding lateral and longitudinal force behaviour from the Delft-tyre model is used while training. The LWPR tuning parameters and the ranges for the three inputs (κ, α, F_z) are set as per Table II. The results of reconstruction are shown in Fig. 2 and Fig. 3.

TABLE II
PARAMETERS FOR COMBINED FORCE RECONSTRUCTION

Parameter	Value	Unit
Normal load	[2000 8000]	N
Longitudinal slip	[-0.5 0.5]	[-]
Slip angle	[-15 15]	deg
Initial distance metric (LWPR)	[1e+3 0 0; 0 1e+3 0; 0 0 5e+1]	[-]

D. Online learning

To analyse the online learning capabilities of the LWPR method for tyre force reconstruction, a badly trained model as shown in Fig. 4 is used to train the LWPR learning module before learning commences. The model is then updated online with the sensing force information (based on the Delft-tyre

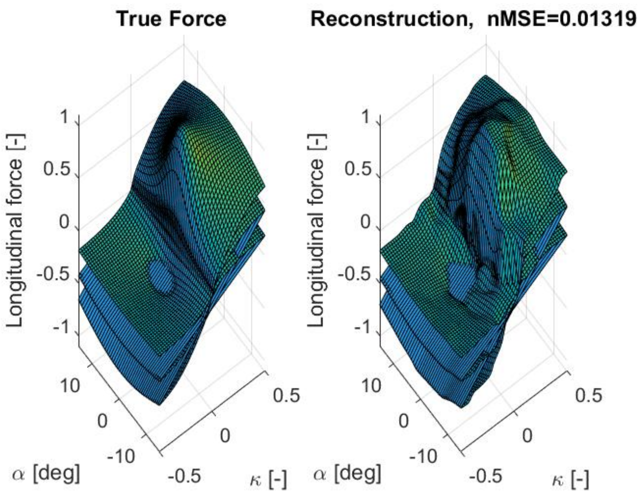

Fig. 2. Longitudinal force reconstruction $F_x(\kappa, \alpha, F_z)$

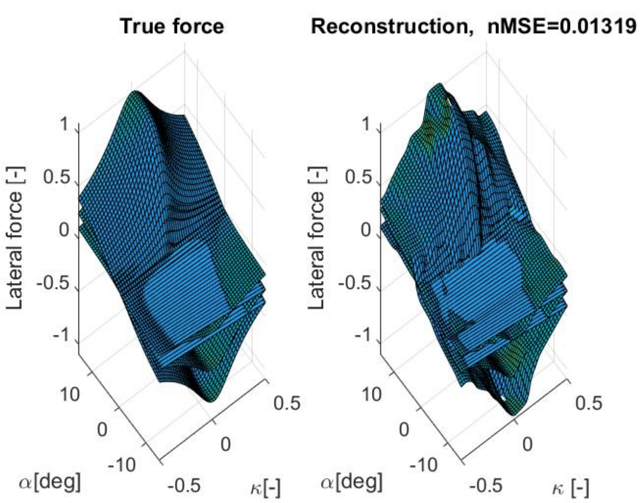

Fig. 3. Lateral force reconstruction $F_y(\kappa, \alpha, F_z))$

model) corresponding to the slip angle range used for training and the predicted result is compared to the badly trained model without adaptation or learning. The simulation parameters used can be seen in Table III. It can be inferred from Fig. 4 that the LWPR algorithm is able to learn the true tyre behaviour online when initialized with a poor tyre model. To assess real-time capability, simulation was conducted using dSPACE real-time (DS1006) machine with IPG/CarMaker HIL.

A significant improvement in tyre force reconstruction, due to online learning, can be inferred from Table IV.

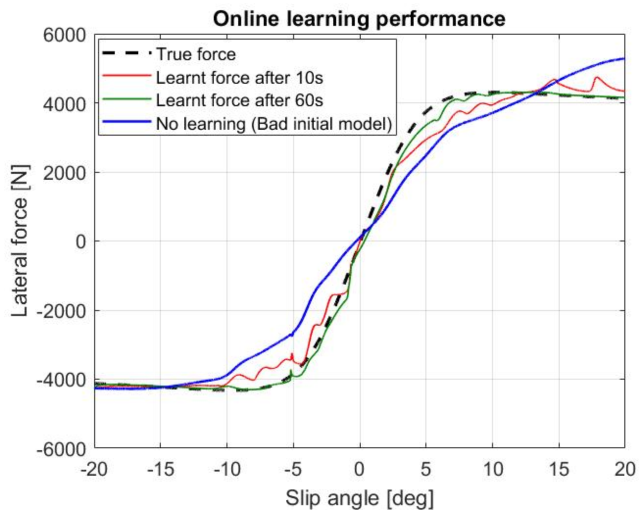

Fig. 4. Online learning performance of LWPR module

TABLE III
PARAMETERS FOR ONLINE LEARNING OF LATERAL FORCE

Parameter	Value	Unit
Normal load	5000	N
Initial distance metric (LWPR)	5000	[-]
Initial component-wise learning rate (LWPR)	40	[-]
Pre-factor of smoothness penalty (LWPR)	0.01	[-]
True force data input rate	100	Hz

TABLE IV
AVERAGE ERROR DURING ONLINE LEARNING OF LATERAL FORCE

Learning duration [s]	Average Error [N]
10	523.1
60	180.0

III. DISCUSSION

The paper discusses various methods for tyre force reconstruction and summarizes three major directions: tyre-model-based, tyre-model-free, and sensor-based approaches. Besides the state-of-the-art overview, the application of a new method named as locally weighted projection regression is considered for tyre force reconstruction.

The main advantage of the considered method is a capability to perform learning through both offline and online training. Among various other techniques, a common approach is the use of neural networks to estimate tyre forces [37]. However, as stated in [36], neural networks are prone to *catastrophic forgetting* which can be described as the tendency to forget old data when fed new data from another distribution. Being immune to this problem, LWPR is an interesting alternative. The simulation results demonstrate that the proposed method can be effectively used to learn steady-state tyre characteristics with a high accuracy as well as real-time feasibility.

A possible application of this algorithm is in autonomous racing. Since it is difficult to predict the tire behavior during the race, online learning of the tyre properties can be considered [38]. Throughout the course of the race, the algorithm can continuously update the tyre model in the controller with sensor data, thereby making it adaptive to changing tire behaviour. This is a potential method of improving tracking performance and minimizing lap time for autonomous racing.

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APPENDIX

Algorithm 1 Incremental PLS

Given : A training point (x, y)
Update the means of input and output :
$x_0^{n+1} = \frac{\lambda W^n x_0^n + wx}{W^{n+1}}$
$\beta_0^{n+1} = \frac{\lambda W^n \beta_0^n + wy}{W^{n+1}}$
where $W^{n+1} = \lambda W^n + w$ and $x_0^0 = u_i^0 = \beta_0^0 = W^0 = 0$
Update the local model :
1. Initialize : $\mathbf{z} = x$,$res_1 = y - \beta_0^{n+1}$
2. For $i = 1 : r$
(i) $u_i^{n+1} = \lambda u_i + wzres_i$
(ii) $s = z^T u_i^{n+1}$
(iii) $SS_i^{n+1} = \lambda SS_i^n + ws^2$
(iv) $SR_i^{n+1} = \lambda SR_i^n + wsres_i$
(v) $SZ_i^{n+1} = \lambda SZ_i^n + wzs$
(vi) $\beta_i^{n+1} = \frac{SR_i^{n+1}}{SS_i^{n+1}}$
(vii) $p_i^{n+1} = \frac{SZ_i^{n+1}}{SS_i^{n+1}}$
(viii) $z = z - sp_i^{n+1}$
(ix) $res_{i+1} = res_i - s\beta_i^{n+1}$
(x) $MSE = \lambda MSE_i^n + wres_{i+1}^2$
Predicting with novel data :
Initialize : $y = \beta_0, z = x - x_0$
For i = 1:k
(i) $s = u_i^T z$
(ii) $y = y + \beta_i s$
(iii) $z = z - sp_i^n$

SS,SR and *SZ* are memory terms that help perform univariate regression using recursive least squares as shown in step (vi) in the model update. Step (vii) helps regress the projection from the current input data **z** and the current projected data s. This ensures that u_{i+1} is orthogonal to u_i. There are two important properties of the local projection scheme. Firstly, if we have statistically independent input variables, PLS takes only a single iteration to find the optimal projection direction u_i. This corresponds to the gradient of the locally linear function to be approximated. Secondly, step (i) in the model update in algorithm 1 ensures that the projection direction is chosen by correlating the input and output data. This results in the automatic exclusion of input dimensions that do not contribute to the output. Finally, since the univariate regressions will never be singular, there is no concern of numerical problems in PLS.

Algorithm 2 Distance metric update

$D = M^T M$, where M is upper triangular
$M^{n+1} = M^n - \alpha \frac{\delta J}{\delta M}$ where the cost function to be minimized is chosen to be,
$J = \frac{1}{W} \sum_{i=1}^{M} \sum_{k=1}^{r} \frac{w_i res_{k+1,i}^2}{1 - w_i \frac{s_{k,i}}{s_k^T W s_k}} + \gamma \sum_{i,j=1}^{N} D_{ij}^2$

The first term in the cost function represents the mean leave-one-out cross-validation error of the local model. The second term is a penalty term which ensures that the receptive fields do not shrink in case of huge amounts of training data.

Algorithm 3 LWPR Outline

1. Initialize the LWPR with no receptive fields
2. **For** every new training sample (x, y)
- **For** $k = 1 : RF$
(i) Calculate the activation from eq. (1)
(ii) Update according to algorithms 1 and 2
- **End**
- **If** no linear model was activated by more than w_{gen}, create a new RF with $r = 2, c = x, D = D_{def}$
- **End**
End

The major assignment within the LWPR framework consists of determining the number of local models k, computing the regression coefficient β_k and the weight w_k for the k^{th} locally linear model. Additionally, it consists of regulating the local model's receptive field. In the algorithm 3, a threshold w_{gen} is predefined. This determines when to create new receptive fields. The closer this value is to 1, the more overlap local models will have, but will be more costly to compute. The distance metric D is initialized to D_{def} and is usually diagonal.